# Lezione 4

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## 1 Retta

$ax + by + c = 0$ è una forma implicita, una condizione della retta. Si tratta di una relazione.

$y = -\frac{a}{b}x - \frac{c}{b}$ è una forma esplicita, ma funziona solo per $b \neq 0$ $\Rightarrow$ non permette di rappresentare rette verticali.

Una retta nello spazio deve essere l'intersezione di 2 piani.
Vogliamo quindi:

- che sia esplicita
- che rappresenti tutte le rette
- che sia valida in 2D e in 3D

  $\Downarrow$

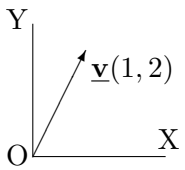
- vettori parametrici (vettore le cui componenti sono funzioni di uno o più parametri)
- retta passante per O ($\underline{\mathbf{v}} = \underline{\mathbf{p}}t \quad t \in \mathbb{R}$)
- retta generica ($\underline{\mathbf{v}} = \underline{\mathbf{p}}t + \underline{\mathbf{q}}$)

### 1.1

$$\underline{\mathbf{v}} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \underline{\mathbf{v}} = \begin{bmatrix} g_x(t) \\ g_y(t) \\ g_z(t) \end{bmatrix} \qquad t \text{ è il "parametro"}$$

$$\underline{\mathbf{v}} = \begin{bmatrix} t^2 \\ 2t^2 \\ 3t^3 \end{bmatrix} \quad t \in \mathbb{R}$$

### 1.2




$$\underline{\mathbf{V}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$\underline{\mathbf{V}} = \begin{bmatrix} t \\ 2t^2 \end{bmatrix}$ $\quad \underline{\mathbf{V}} = (t, 2t^2)$ $\quad$ al variare di $t$ ottengo un'infinità di punti

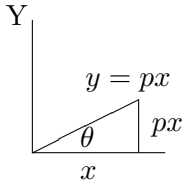
Il luogo geometrico dei punti le cui coordinate sono espresse dalle componenti del vettore parametrico.

$$\underline{\mathbf{V}} = \begin{bmatrix} t \\ 2t^2 \end{bmatrix} \quad \rightarrow \quad \begin{cases} x = t \\ y = 2t^2 \end{cases} \quad , t \in \mathbb{R}$$

## 2 Rappresentazione parametrica vettoriale

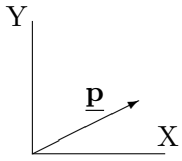
### 2.1 implicita

$(b \neq 0) \quad y = px$ $\qquad p$ è il coefficiente angolare ovvero la pendenza della retta.




$p = \tan\theta \qquad \frac{px}{x} = \tan\theta$

### 2.2 esplicita



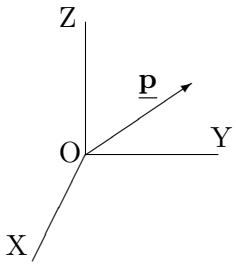

$$\underline{\mathbf{p}} = \begin{bmatrix} p_x \\ p_y \end{bmatrix} \qquad \underline{\mathbf{v}} = t\underline{\mathbf{p}} \quad , t \in \mathbb{R} \quad \rightarrow \quad t\underline{\mathbf{p}} = \begin{bmatrix} t\underline{\mathbf{p}}_x \\ t\underline{\mathbf{p}}_y \end{bmatrix}$$

## 2.3

$V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ , $V_2 = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$ sono due parametrizzazioni diverse della stessa retta.

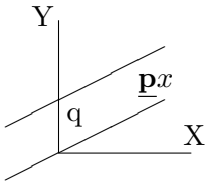
$$\underline{\mathbf{V}} = t\underline{\mathbf{p}} = \begin{bmatrix} t\underline{\mathbf{p}}_x \\ t\underline{\mathbf{p}}_y \end{bmatrix} = \begin{cases} x = t\underline{\mathbf{p}}_x \\ y = t\underline{\mathbf{p}}_y \end{cases} = \begin{cases} t = \frac{x}{\underline{\mathbf{p}}_x} \\ y = \frac{p_y}{p_x} x \end{cases}$$

## 2.4



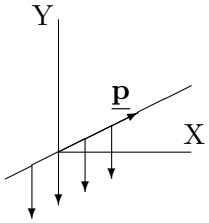

$$\underline{\mathbf{p}} = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \qquad \underline{\mathbf{v}} = \underline{\mathbf{p}}t \qquad \underline{\mathbf{v}} = \begin{bmatrix} \underline{\mathbf{p}}_x t \\ \underline{\mathbf{p}}_y t \\ \underline{\mathbf{p}}_z t \end{bmatrix}$$

## 2.5 non passa per 0




$y = px + q$

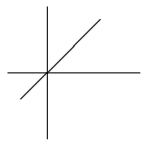
## 2.6 passa per 0



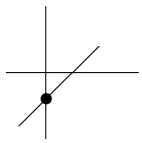

$\underline{\mathbf{v}} = \underline{\mathbf{p}}t + \underline{\mathbf{q}}$

### 2.7

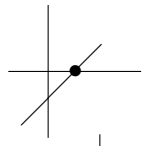
$\underline{\mathbf{v}}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} t$



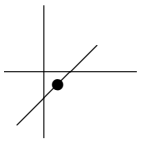
$\underline{\mathbf{v}}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} t + \begin{bmatrix} 0 \\ -1 \end{bmatrix}$



$\underline{\mathbf{v}}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} t + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$



$\underline{\mathbf{v}}_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} t + \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix}$



$\underline{\mathbf{v}}_1, \underline{\mathbf{v}}_2, \underline{\mathbf{v}}_3$ sono parametrizzazioni diverse della stessa retta.

### 2.8

$$\underline{\mathbf{v}} = \underline{\mathbf{p}}t + \underline{\mathbf{q}} = \begin{bmatrix} p_x t \\ p_y t \\ p_z t \end{bmatrix} + \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = \begin{cases} x = p_x t + q_x \\ y = p_y t + q_y \\ z = p_z t + q_z \end{cases}$$

## 3

$\underline{\mathbf{v}} = \underline{\mathbf{p}}t + \underline{\mathbf{q}}$
$y = xp + q$

### 3.1

$P_1(1, 2) \qquad P_2(3, 6)$

$\begin{cases} 2 = p + q \\ 6 = 3p + q \end{cases} \quad 4 = 2p \quad p = 2$

$2 = 2 + q \quad \rightarrow \quad q = 0$
$y = 2x$

### 3.2

$P_1(1,2) \qquad P_2(3,7)$

$$\begin{cases} 2 = p + q \\ 7 = 3p + q \end{cases} \qquad \begin{matrix} 5 = 2p & \rightarrow & p = \frac{5}{2} \\ 2 = \frac{5}{2} + q & \rightarrow & q = -\frac{1}{2} \end{matrix}$$

### 3.3

$$\underline{\mathbf{p}}_1 = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \qquad \underline{\mathbf{p}}_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$\underline{\mathbf{p}} = \underline{\mathbf{p}}_1 - \underline{\mathbf{p}}_2 = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

$$\underline{\mathbf{v}} = \begin{bmatrix} 3 \\ -3 \end{bmatrix} t + \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$\begin{cases} x = 3t + 4 \\ y = -3t + 1 \end{cases} \qquad \begin{cases} 3t = x - 4 \\ y = 4 - x + 1 \end{cases} \qquad y = -x + 5$$

### 3.4

$$\underline{\mathbf{P}}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \underline{\mathbf{P}}_2 = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$$

$$P = \underline{\mathbf{P}}_1 - \underline{\mathbf{P}}_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

$$\underline{\mathbf{v}} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} t + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

## 4

Se ho due rette, quando sono $\perp$, $\parallel$ o coincidenti?

$\underline{\mathbf{V}} = \underline{\mathbf{p}}t + \underline{\mathbf{q}}$
$\underline{\mathbf{W}} = \underline{\mathbf{m}}t + \underline{\mathbf{n}}$

## 4.1 parallele

$\underline{\mathbf{p}} = \alpha \underline{\mathbf{m}} \quad , \alpha \in \mathbb{R}$ se esiste $\alpha$ allora sono parallele

### 4.1.1

$\underline{\mathbf{p}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \underline{\mathbf{m}} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} 3 \\ 7 \end{bmatrix}$ non esiste $\alpha \Rightarrow$ non sono $\parallel$

### 4.1.2

$\underline{\mathbf{p}} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \qquad \underline{\mathbf{m}} = \begin{bmatrix} -1 \\ 4 \\ -2 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix} \qquad \alpha = \frac{1}{2} \qquad \Rightarrow$ sono $\parallel$

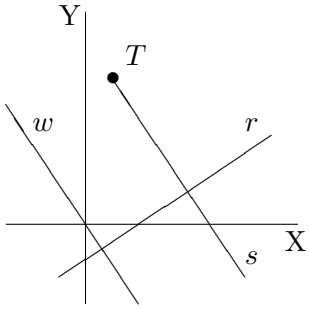
( $\parallel \Rightarrow \underline{\mathbf{p}} = \alpha \underline{\mathbf{m}}$ )

## 4.2 perpendicolari in 2D

$\underline{\mathbf{V}} = \underline{\mathbf{p}} t + \underline{\mathbf{q}}$

$\underline{\mathbf{W}} = \underline{\mathbf{m}} t' + \underline{\mathbf{n}}$

$\perp \quad \Rightarrow \quad \underline{\mathbf{p}} \cdot \underline{\mathbf{m}} = 0$




### 4.2.1

$r : \underline{\mathbf{v}} = \begin{bmatrix} p_x \\ p_y \end{bmatrix} t + \begin{bmatrix} q_x \\ q_y \end{bmatrix} \qquad \begin{bmatrix} p_x \\ p_y \end{bmatrix}$ è noto

$\underline{\mathbf{t}} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$
$T(t_x, t_y)$
$s : \underline{\mathbf{w}} = \begin{bmatrix} m_x \\ m_y \end{bmatrix} t' + \begin{bmatrix} n_x \\ n_y \end{bmatrix}$

sono $\perp$ se $\underline{\mathbf{p}} \cdot \underline{\mathbf{m}} = 0$

$p_x m_x + p_y m_y = 0$

per $m_y = 1 \quad \rightarrow \quad p_x m_x + p_y = 0 \quad m_x = -\frac{p_y}{p_x}$

$\underline{\mathbf{m}} = \begin{bmatrix} -\frac{p_y}{p_x} \\ 1 \end{bmatrix} \qquad \frac{1}{-p_y/p_x} = -\frac{p_x}{p_y}$

$\underline{\mathbf{w}} = \underline{\mathbf{m}} t' + \underline{\mathbf{t}}$

### 4.2.2

$r : \underline{\mathbf{v}} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} t$
$s \perp r$ passante per $(1, 1)$

$\underline{\mathbf{p}} \cdot \underline{\mathbf{m}} = 2m_x + 1m_y = 0$
$m_y = 1$

$2m_x = -1 \quad \rightarrow \quad m_x = -\frac{1}{2}$

$\underline{\mathbf{m}} = \begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix} \qquad (m$ è $\perp$ a $\underline{\mathbf{v}})$

$s : w = \begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix} t' + \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

## 4.3 perpendicolari in 3D

$r : \underline{\mathbf{v}} = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} t + \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix}$

$\underline{\mathbf{p}} \cdot \underline{\mathbf{m}} = 0$ (ci sono 2 gradi di libertà)

$p_x m_x + p_y m_y + p_z m_z = 0$

### 4.3.1

$\underline{\mathbf{p}} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ $\quad 1m_x + (-2)m_y + 3m_z = 0$

$m_y = 1$
$m_z = 1]$
$m_x = -2 + 3 = 0 \qquad m_x = -1$

$\underline{\mathbf{m}} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

prova: $-1 - 2 - 3 = 0$ punto a scelta $(-1, -2, -3)$

$s : w = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} t' + \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$

### 4.3.2

$\underline{\mathbf{v}} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} t$ (noto)

$\underline{\mathbf{w}} = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} t' + \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ (noto)

sono $\parallel$ ? $\quad \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$ $\quad \alpha$ non esiste

sono $\perp$ ? $\quad 1 \cdot 0 + 2 \cdot (-1) + 1 \cdot 2 = 0 - 2 + 2 = 0$ $\quad$ sì, sono $\perp$

sono incidenti?

$$\begin{cases} x = t \\ y = zt \\ z = t \end{cases} \qquad \begin{cases} x = & 0 \cdot t' + 1 \\ y = & -1 \cdot t' + 0 \\ z = & 2 \cdot t' + (-1) \end{cases} \qquad \begin{cases} x = 1 \\ y = -t' \\ z = 2t' - 1 \end{cases}$$

$$\begin{cases} \bar{t} = 1 \\ 2\bar{t} = -\bar{t}' \\ z = t \end{cases} \qquad \begin{cases} \bar{t} = & 1 \\ 2 = & -\bar{t}' \\ \bar{t} = & 2\bar{t}' - 1 \end{cases} \qquad \begin{matrix} (\bar{t}, \bar{t}') \\ (1, -2) \\ \rightarrow 1 = -4 - 1 \quad \Rightarrow \text{non esiste} \end{matrix}$$

$\Rightarrow$ le rette sono perpendicolari ma NON incidenti.

### 4.3.3

$\underline{\mathbf{p}} \cdot \underline{\mathbf{m}} = 0 \quad \Rightarrow \quad \perp$

$$\underline{\mathbf{v}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} t + \begin{bmatrix} 3 \\ 2 \end{bmatrix} \qquad \begin{cases} x = t + 3 \\ y = 2t + 2 \end{cases}$$

$$\underline{\mathbf{w}} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} t' + \begin{bmatrix} 0 \\ -3 \end{bmatrix} \qquad \begin{cases} x = -2t' \\ y = t' - 3 \end{cases}$$

$$\begin{cases} t + 3 = -2t' \\ 2t + 2 = t' - 3 \end{cases}$$

$t' = 2t + 5$
$t + 3 = -4t - 10 \qquad 5t = -13 \qquad t = -\frac{13}{5}$

### 4.3.4

$$\underline{\mathbf{v}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} t \qquad \begin{cases} x = t \\ y = 2t \end{cases}$$

$$\underline{\mathbf{w}} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} t' + \begin{bmatrix} -2 \\ -4 \end{bmatrix} \qquad \begin{cases} x = 2t' - 2 \\ y = 4t' - 4 \end{cases}$$

$\begin{cases} t = 2t' - 2 \\ 2t = 4t' - 4 \end{cases}$ hanno infinite soluzioni/punti in comune $\Rightarrow$ sono coincidenti.