Lezione 15

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1 Equazione di una retta

1.1

$\underline{P}_0 \qquad \underline{P}_1 \quad t \in \mathbb{R}$

$r : (\underline{P}_1 - \underline{P}_0)t + \underline{P}_0 \qquad 0 \leq t \leq 1$

(prendo in considerazione solo il segmento che va da $\underline{P}_0$ a $\underline{P}_1$

$r(0) = \underline{P}_0 \qquad r(q) = \underline{P}_1$

$r(t) = \underline{P}_1 t - \underline{P}_0 t + \underline{P}_0 = \underline{P}_0(1-t) + \underline{P}_1(t)$

1.2

$\underline{r}(t) = \underline{P}_1 t - \underline{P}_0 t + \underline{P}_0 \quad = \quad \underline{P}_0(1-t) + \underline{P}_1 t$

2 Bezier

$$\begin{array}{ccc}
\underline{P}_0(1-t) + \underline{P}_1 t & & \underline{P}_1(1-t) + \underline{P}_2 t \\
\downarrow & & \downarrow \\
P_0' & & P_1' \\
\searrow & & \swarrow \\
. & P_0'(1-t) + P_1' t &
\end{array}$$

$[\underline{P}_0(1-t) + \underline{P}_1 t]\,(1-t) + [\underline{P}_1(1-t) + \underline{P}_2 t]\,t =$
$\underline{P}_0(1-t)^2 + 2\underline{P}_1 t(1-t) + \underline{P}_2 t^2$

3 Tangente ad una curva di Bezier

$\underline{r}(t) = \underline{P}_0(1-t)^2 + \underline{P}_1 2t(1-t) + \underline{P}_2 t^2$

Svolgo i calcoli e derivo:

$g(t) = (1-t)^2 = 1 - 2t + t^2$
$g'(t) = 0 \cdot 2 + 2t = 2(t-1)$

$g(t) = t(1-t) = t - t^2$
$g'(t) = 1 - 2t$

Ne risulta:

$\underline{r}'(t) = \underline{P}_0 2(t-1) + 2\underline{P}_1(1-2t) + \underline{P}_2 2t$

$\Downarrow$

$\underline{r}'(0) = -2\underline{P}_0 + 2\underline{P}_1 = 2(\underline{P}_1 - \underline{P}_0)$

$\underline{r}'(1) = -2\underline{P}_1 + 2\underline{P}_2 = 2(\underline{P}_2 - \underline{P}_1)$

4 Curve di Bezier raccordate

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$2(\underline{P}_2 - \underline{P}_1) = 2(\underline{P}'_1 - \underline{P}'_0)$
$(\underline{P}_2 - \underline{P}_1) = (\underline{P}'_1 - \underline{P}_2$

$$\underline{P}'_1 = 2(\underline{P}_2 - \underline{P}_1)$$

4.1 Raccordi curve di Bezier di grado 2

$\underline{P}_0 = (0,0) \qquad \underline{P}'_0 = (2,0)$
$\underline{P}_1 = (1,2) \qquad \underline{P}'_1 = (x_1, y_1)$
$\underline{P}_2 = (2,0) \qquad \underline{P}'_2 = (4,0)$

$\underline{P}'_1 = 2\begin{bmatrix}2\\0\end{bmatrix} - \begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}3\\-2\end{bmatrix}$

4.2 Raccordi curve di Bezier di grado 3

$\underline{r} = \underline{P}_0(1-t)^3 + \underline{P}_1 3(1-t)^2 t + \underline{P}_2 3(1-t)^2 + \underline{P}_3 t^3$

$\underline{r}' = -\underline{P}_0 3(t-1)^2 + \underline{P}_1 3(1-4t+3t^2) + \underline{P}_2 3(2t-3t^2) + \underline{P}_3 3t^2$

$\underline{r}'(0) = -3\underline{P}_0 + 3\underline{P}_1 = 3(\underline{P}_1 - \underline{P}_0)$
$\underline{r}'(1) = -3\underline{P}_2 + 3\underline{P}_3 = 3(\underline{P}_3 - \underline{P}_2)$

$3(\underline{P}_3 - \underline{P}_2) = 3(\underline{P}_1 - \underline{P}_3)$

$$\underline{P}_1' = 2(\underline{P}_3 - \underline{P}_2)$$

4.3

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$2(\underline{P}_0 - \underline{P}_1') = 3(\underline{P}_1 - \underline{P}_0)$

$2\underline{P}_0 - 2\underline{P}_1' = 3\underline{P}_1' - 3\underline{P}_0 \quad \Rightarrow \quad 5\underline{P}_0 - 3\underline{P}_1 = \underline{P}_1'$

4.3.1 esempio

Dati: $\underline{P}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \underline{T}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
$\underline{P}_3 = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \qquad \underline{T}_3 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Trovare $\underline{P}_1$ e $\underline{P}_2$.

$\underline{T}_0 = 3(\underline{P}_1 - \underline{P}_0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \Rightarrow \quad 3\underline{P}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \rightarrow \quad \underline{P}_1 = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$

$\underline{T}_3 = 3(\underline{P}_3 - \underline{P}_2) = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad \Rightarrow \quad 3\underline{P}_2 = 3\underline{P}_3 - \begin{bmatrix} -1 \\ 1 \end{bmatrix} = 3\begin{bmatrix} 4 \\ 0 \end{bmatrix} - \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad \rightarrow$
$\rightarrow \quad \underline{P}_2 = \begin{bmatrix} \frac{13}{3} \\ -\frac{1}{3} \end{bmatrix}$